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ROTOR MACHINERY DYNAMICS

Practical Trainings

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Contents

Topic 1. The fundamentals of rotor dynamics

A simple conservative single-mass model of rotor dynamics. Direct synchronous precession. A self-balancing phenomenon. Equation of a motion for a single-mass model of the unbalanced rotor. Considering the external friction. Kinematics of rotating rotor’s precessions. Equation of rotor dynamics considering the anisotropy of elastic forces. Loss of dynamic stability. Forced oscillations of a balanced horizontal rotor. Kinematics of a horizontal rotor. Equation of rotor dynamics considering the impact of a liquid layer. Influence of the circulating force. Determination of amplitude and phase frequency responses. Dynamic stability of a centrifugal pump’s rotor.

Pr. tr. 1. Dynamic analysis of the conservative single-mass model

1. The amplitude-frequency response for a single-mass model of rotor dynamics.
2. Determination of the maximum deflection of a rotor at the operating frequency.
3. Evaluation of the total dynamic response at the operating frequency.

The initial data is presented in Table 1.1, where the following parameters are introduced: d – shaft’s diameter, m; L – shaft’s length, m; m_0 – mass of the impeller, kg; n_0 – operating speed, rpm; ρ – density, kg/m^3; E – Young’s modulus, N/m^2; c_s – bearing stiffness, N/m.

Table 1.1 – Initial data for Pr. tr. 1

Var.	d, m	L, m	m_0, kg	n_0, rpm	ρ, kg/m^3	E, 10^{11} N/m^2	c_s, 10^8 N/m
1	0.015	0.6	10	1500	7800	2.00	1
2	0.020	0.7	15	3000	7850	2.05	2
3	0.025	0.8	20	4500	7800	2.10	3
4	0.030	0.9	25	6000	7850	2.00	4
5	0.035	1.0	30	7500	7800	2.05	5
6	0.040	1.1	35	9000	7850	2.10	1
7	0.045	1.2	10	1050	7800	2.00	2
8	0.050	1.3	15	1500	7850	2.05	3
9	0.015	1.4	20	3000	7800	2.10	4
10	0.020	1.5	25	4500	7850	2.00	5
11	0.025	0.6	30	6000	7800	2.05	1
12	0.030	0.7	35	7500	7850	2.10	2
13	0.035	0.8	10	9000	7800	2.00	3
14	0.040	0.9	15	1050	7850	2.05	4
15	0.045	1.0	20	1500	7800	2.10	5
16	0.050	1.1	25	3000	7850	2.00	1
17	0.015	1.2	30	4500	7800	2.05	2
18	0.020	1.3	35	6000	7850	2.10	3
19	0.025	1.4	10	7500	7800	2.00	4
20	0.030	1.5	15	9000	7850	2.05	5

The calculation technique is as follows:

1) the amplitude-frequency response for a single-mass model of rotor dynamics:

– shaft mass, kg:

$$m_s = \rho \frac{\pi d^2}{4} L; \tag{1.1}$$

– the equivalent mass of the rotor, kg:

$$m_e = m_0 + m_s; \tag{1.2}$$

– the cross-sectional moment of inertia, m^4:

$$I = \frac{\pi d^4}{4}; \tag{1.3}$$

– bending stiffness of the simply-supported shaft, N/m:

$$c_b = \frac{48EI}{L^3}; \tag{1.4}$$

– equivalent stiffness of the system "rotor – bearings", N/m:

$$c_e = \frac{2c_b c_s}{c_b + 2c_s}; \tag{1.5}$$

– eigenfrequency, rad/s:

$$\omega_{cr} = \sqrt{\frac{c_e}{m_e}}; \tag{1.6}$$

– critical speed, rpm:

$$n_{cr} = \frac{30}{\pi}\omega_{cr}; \tag{1.7}$$

– operating speed, rad/s:

$$\omega_0 = \frac{\pi n_{cr}}{30}; \tag{1.8}$$

– amplitude-frequency response, m:

$$A(\omega) = \frac{m_e \omega^2 e}{|c_e - m_e \omega^2|}, \tag{1.9}$$

where ω – rotational speed, rad/s; e – permissible eccentricity, m;

2) determination of the maximum deflection of a rotor at the operating frequency:

– amplitude at the operating speed, m:

$$A_0 = \frac{m_e \omega_0^2 e}{|c_e - m_e \omega_0^2|}, \qquad (1.10)$$

3) evaluation of the total dynamic response at the operating frequency:

– the total dynamic reaction force, N:

$$R_0 = c_e A_0. \qquad (1.11)$$

Pr. tr. 2. Dynamic analysis of a single-mass model considering external friction

1. Amplitude and phase-frequency responses for a single-mass model.
2. Determination of the maximum deflection of a rotor at the critical frequency.
3. Evaluation of the total dynamic response at the critical frequency.

The initial data is similar to Pr. tr. 1. However, damping is additionally considered (Table 1.2), where $\bar{b}$ – dimensionless damping factor.

Table 1.2 – Additional data for Pr. tr. 2

Var.	1	2	3	4	5	6	7	8	9	10
$\bar{b}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Var.	11	12	13	14	15	16	17	18	19	20
$\bar{b}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

The calculation technique is as follows:

1) amplitude and phase-frequency responses for a single-mass model:

– determination of eigenfrequency by formulas (1.1)–(1.6);

– damping factor, N·s/m:

$$b = m_e \omega_{cr} \bar{b}; \qquad (1.12)$$

– amplitude-frequency response, m:

$$A(\omega) = \frac{m_e \omega^2 e}{\sqrt{(c_e - m_e \omega^2)^2 + (b\omega)^2}}; \qquad (1.13)$$

– phase-frequency response, rad:

$$\varphi(\omega) = \begin{cases} arctg\left(\frac{b\omega}{c_e - m_e\omega^2}\right), if\ \omega < \omega_0; \\ \frac{\pi}{2}, if\ \omega = \omega_0; \\ \pi + arctg\left(\frac{b\omega}{c_e - m_e\omega^2}\right), if\ \omega > \omega_0; \end{cases} \qquad (1.14)$$

2) determination of the maximum deflection of a rotor at the critical frequency, m:

$$A_{max} = \frac{\sqrt{m_e c_e}}{b} e; \qquad (1.15)$$

3) evaluation of the total dynamic response at the critical frequency, N:

$$R_{max} = c_e A_{max}. \tag{1.16}$$

Pr. tr. 3. Dynamic analysis of a single-mass model for a double rigidity rotor

1. Determination of the critical frequencies of the first kind.
2. Detuning from the resonanse.
3. The dynamic stability conditions for the rotor at the operating frequency.
4. Determination of the critical frequency of the second kind.
5. Relative and absolute trajectories of the mass center.

The initial data is similar to Pr. tr. 1. However, the damping factor is additionally considered (Table 1.3), where δ_a – anisotropy factor; δ_{cr} – detuning from the resonance.

Table 1.3 – Additional data for Pr. tr. 3

Var.	δ_a	δ_{cr}	Var.	δ_a	δ_{cr}
1	0.05	0.25	11	0.25	0.30
2	0.10	0.30	12	0.30	0.35
3	0.15	0.35	13	0.05	0.40
4	0.20	0.40	14	0.10	0.45
5	0.25	0.45	15	0.15	0.50
6	0.30	0.50	16	0.20	0.55
7	0.05	0.55	17	0.25	0.60
8	0.10	0.60	18	0.30	0.65
9	0.15	0.65	19	0.05	0.25
10	0.20	0.25	20	0.10	0.30

The calculation technique is as follows:

1) determination of the critical frequencies of the first kind:

– calculation of equivalent mass by formulas (1.1)–(1.2);

– calculation of the cross-sectional moment of inertia by formula (1.3);

– determination of the bending double-rigidity stiffness, N/m:

$$c_{by} = \frac{48EI}{L^3};\, c_{bx} = (1 - \delta_a)c_{by}; \tag{1.17}$$

– determination of the equivalent double-rigidity stiffness, N/m:

$$c_{ex} = \frac{2c_{bx}c_s}{c_{bx}+2c_s};\; c_{ey} = \frac{2c_{by}c_s}{c_{by}+2c_s}; \tag{1.18}$$

– evaluation of the critical frequencies of the 1st kind, rad/s:

$$p_1 = \sqrt{\frac{c_{ex}}{m_e}};\; p_2 = \sqrt{\frac{c_{ey}}{m_e}}; \tag{1.19}$$

2) detuning from the resonance:

– operating frequency, rad/s:

$$\omega_0 = (1 - \delta_{cr})p_2; \tag{1.20}$$

3) the dynamic stability conditions for the rotor at the operating frequency:

– calculation of following imaginary parameters, rad/s:

$$p_I = \sqrt{\frac{-(p_1^2+p_2^2+2\omega^2)+\sqrt{\left(p_1^2-p_2^2\right)^2+8\omega^2(p_1^2+p_2^2)}}{2}};$$
$$p_{II} = \sqrt{\frac{-(p_1^2+p_2^2+2\omega^2)-\sqrt{\left(p_1^2-p_2^2\right)^2+8\omega^2(p_1^2+p_2^2)}}{2}}; \tag{1.21}$$

4) determination of the critical frequency of the second kind, rad/s:

$$\omega_{cr_2} = \frac{p_1 p_2}{\sqrt{2\left(p_1^2 + p_2^2\right)}}; \qquad (1.22)$$

5) relative and absolute trajectories of the mass center:
– semiaxes of the elliptical trajectory, m:

$$\binom{A}{B} = -\begin{bmatrix} p_1^2 - 2\omega_0^2 & 2\omega_0^2 \\ 2\omega_0^2 & p_2^2 - 2\omega_0^2 \end{bmatrix} \binom{1}{1} g, \qquad (1.23)$$

where g = 9.81 m/s^2 – acceleration of gravity;
– geometrical components of the trajectory, m:

$$\Delta = \frac{A+B}{2}; r = \frac{B-A}{2}; \qquad (1.24)$$

– coordinates of the trajectory, m:

$$\begin{cases} x(t) = r\sin(2\omega_0 t); \\ y(t) = \Delta - r\cos(2\omega_0 t). \end{cases} \qquad (1.25)$$

where t – time, s.

Pr. tr. 4. Dynamics of a single-mass model for a centrifugal pump rotor

1. Determination of the critical frequency.
2. Amplitude and phase-frequency characteristics.
3. The dynamic stability of the centrifugal pump rotor.

The initial data is similar to Pr. tr. 1, 2. However, external damping is additionally considered (Table 1.4), where $\Delta\bar{b}$ – dimensionless external damping factor, N·s/m.

Table 1.4 – Additional data for Pr. tr. 4

Var.	1	2	3	4	5	6	7	8	9	10
$\Delta\bar{b}$	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
Var.	11	12	13	14	15	16	17	18	19	20
$\Delta\bar{b}$	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1

The calculation technique is as follows:

1) determination of the critical frequency:

– determination of eigenfrequency by formulas (1.1)–(1.6);

2) amplitude and phase-frequency characteristics:

– determination of damping factor by formula (1.12);

– external damping factor, N·s/m:

$$\Delta b = m_e \omega_{cr} \Delta\bar{b}; \qquad (1.26)$$

– coefficient of the circulating force, N/m:

$$q(\omega) = \frac{1}{2} b\omega; \qquad (1.27)$$

– amplitude-frequency response, m:

$$A(\omega) = \frac{m_e \omega^2 e}{\sqrt{(c_e - m_e\omega^2)^2 + [(b+\Delta b)\omega - q(\omega)]^2}}; \qquad (1.28)$$

– phase-frequency response, rad:

$$\varphi(\omega) = \begin{cases} arctg\left[\frac{(b+\Delta b)\omega - q(\omega)}{c_e - m_e\omega^2}\right], if\ \omega < \omega_0; \\ \frac{\pi}{2}, if\ \omega = \omega_0; \\ \pi + arctg\left[\frac{(b+\Delta b)\omega - q(\omega)}{c_e - m_e\omega^2}\right], if\ \omega > \omega_0; \end{cases} \qquad (1.29)$$

3) the dynamic stability of the centrifugal pump rotor:
– calculation of the operating speed by formula (1.8);
– maximum operating speed of the stability loss, rad/s:

$$\omega_{sl} = 2\left(1 + \frac{\Delta b}{b}\right)\omega_{cr}. \qquad (1.30)$$

Topic 2. Study of rotor dynamics by discrete models of oscillations

The primary dependencies. Self-oscillation of a rotor without contact interaction with the stator. Self-oscillating precession of a rotor under contact with the stator. A mathematical model of self-oscillations for a floating ring considering dry friction. Stability and self-oscillations of a single-mass model considering anisotropy of elastic forces. Influence of internal viscous friction on dynamics of a horizontal rotor. Basic approaches to creating discrete models of rotor dynamics. The traditional discrete multi-mass model. Ways to consider the gyroscopic moment of inertia in rotor dynamics. Influence of the gyroscopic moment of inertia on rotor’s critical frequencies. Shape functions of a 2D beam-type finite element. Lagrange equations of the 2nd kind for transverse oscillations of a beam element. Matrix equation of rotor dynamics. Free and forced oscillations of the rotor’s finite element model.

Pr. tr. 5. Self-oscillations of the centrifugal pump rotor

1. Hydrodynamic characteristics of gap seals.
2. Determination of the limiting frequency of the rotor's self-oscillations without contact with the stator.
3. Self-oscillations of the rotor under contact with the stator.
4. The dependence of the self-oscillations amplitude on the rotor speed.

The initial data is similar to Pr. tr. 1, 2, 4. However, the following additional parameters are introduced (Table 2.1): h_0 – the radial gap in the throttle, m; c_0 – initial hydrodynamic stiffness of the throttling gap, N/m; α – stiffness factor; c_c – contact stiffness between the rotor and the stator, N/m; f – friction coefficient.

Table 2.1 – Additional data for Pr. tr. 5

Var.	h_0, 10^{-3} m	c_0, 10^6 N/m	α	c_c, 10^{11} N/m	f
1	0.10	1	0.1	1	0.05
2	0.12	2	0.2	2	0.10
3	0.14	3	0.3	3	0.15
4	0.16	4	0.4	4	0.20
5	0.18	5	0.5	5	0.25
6	0.20	6	0.1	6	0.30
7	0.25	7	0.2	1	0.05
8	0.30	8	0.3	2	0.10
9	0.10	9	0.4	3	0.15
10	0.12	1	0.5	4	0.20
11	0.14	2	0.1	5	0.25
12	0.16	3	0.2	6	0.30
13	0.18	4	0.3	1	0.05

Var.	h_0, 10^{-3} m	c_0, 10^6 N/m	α	c_c, 10^{11} N/m	f
14	0.20	5	0.4	2	0.10
15	0.25	6	0.5	3	0.15
16	0.30	7	0.1	4	0.20
17	0.10	8	0.2	5	0.25
18	0.12	9	0.3	6	0.30
19	0.14	1	0.4	1	0.05
20	0.16	2	0.5	2	0.10

The calculation technique is as follows:

1) hydrodynamic characteristics of gap seals:

– determination of eigenfrequency by formulas (1.1)–(1.6);

– hydrodynamic stiffness of the throttling gap, N/m:

$$c(r) = c_0\left(1 + \frac{3}{4}\alpha^2\frac{r^2}{h_0^2}\right), \tag{2.1}$$

where r – radial displacement, m;

– maximum stiffness of the throttling gap, N/m:

$$c_{max} = c_0\left(1 + \frac{3}{4}\alpha^2\right); \tag{2.2}$$

– initial damping factor, N·s/m:

$$b_0 = m_e\omega_{cr}\bar{b}; \tag{2.3}$$

– damping factor of the throttling gap, N·s/m:

$$b(r) = b_0\left(1 + \frac{3}{2}\frac{r^2}{h_0^2}\right); \tag{2.4}$$

2) determination of the limiting frequency of the rotor's self-oscillations without contact with the stator:

– critical frequency of the rotor, rad/s:

$$\omega_{cr} = \sqrt{\frac{c_e + c_0}{m_e}}; \qquad (2.5)$$

– partial frequency of the sliding bearing, rad/s:

$$\omega_{sb} = \sqrt{\frac{c_0}{m_e}}; \qquad (2.6)$$

– precession frequency, rad/s:

$$\Omega = 0.5\omega_{cr}; \qquad (2.7)$$

– limiting frequency range, rad/s:

$$\omega_{min} = 2\omega_{cr};\ \omega_{max} = 2\omega_{cr}\sqrt{1 + \frac{3}{4}\alpha\left(\frac{\omega_{sb}}{\omega_{cr}}\right)^2}; \qquad (2.8)$$

3) self-oscillations of the rotor under contact with the stator:
– critical frequency of the rotor under contact with the stator, rad/s:

$$\omega_{cr} = \sqrt{\frac{c_e + c_{max}}{m_e}}; \qquad (2.9)$$

– partial frequency of the contact pair, rad/s:

$$\omega_c = \sqrt{\frac{c_c}{m_e}}; \qquad (2.10)$$

– the variation of the dimensionless amplitude:

$$\delta a(\omega) = \frac{\omega^2 - 4\omega_{max}^2}{4\omega_c^2\left(1 + f\frac{m_e}{b}\omega\right)}; \qquad (2.11)$$

4) the dependence of the self-oscillations amplitude on the rotor speed:

– amplitude before contact with the stator, m:

$$A_1(\omega) = \frac{2h_0}{\alpha\sqrt{3}} \frac{\omega_{cr}}{\omega_{sb}} \sqrt{\frac{\omega^2}{4\omega_{cr}^2} - 1}; \qquad (2.12)$$

– amplitude under contact with the stator, m:

$$A_1(\omega) = h_0[1 + \delta a(\omega)]. \qquad (2.13)$$

Pr. tr. 6. Dynamic stability of the floating seal ring

1. The dynamics of the horizontal rotor under internal viscous friction.
2. Characteristics of self-oscillating precession of a floating ring considering dry friction.
3. The stability of a single-mass model considering the anisotropy of elastic forces.
4. Characteristics of self-oscillating precession of the rotor considering dry friction.

The initial data is similar to Pr. tr. 1–5. However, the following additional parameters are introduced (Table 2.2): $\overline{b}_f$ – internal viscous friction ratio, N·s/m; m_r – mass of the floating ring, kg; F_n – Nominal clamping force of the floating ring, N.

Table 2.2 – Additional data for Pr. tr. 6

Var.	1	2	3	4	5	6	7	8	9	10
m_r, kg	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4	0.5	0.6
F_n, 10^3 N	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.1
$\bar{b}_f$	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4	0.1	0.2
Var.	11	12	13	14	15	16	17	18	19	20
m_r, kg	0.7	0.8	0.3	0.4	0.5	0.6	0.7	0.8	0.3	0.4
F_n, 10^3 N	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.1	0.2
$\bar{b}_f$	0.3	0.4	0.1	0.2	0.3	0.4	0.1	0.2	0.3	0.4

The calculation technique is as follows:

1) the dynamics of the horizontal rotor under internal viscous friction:

– determination of eigenfrequency by formulas (1.1)–(1.6);

– determination of damping factor by formula (1.12);

– calculation of internal damping factor, N·s/m:

$$b_f = m_e \omega_{cr} \bar{b}_f; \qquad (1.14)$$

– damping coefficient, s^{-1}:

$$k = \frac{b}{m_e}; \qquad (2.15)$$

– internal damping coefficient, s^{-1}:

$$\delta = \frac{b_f}{m_e}; \qquad (2.16)$$

– amplitude of the static displacement, m:

$$A_s(\omega) = \frac{g}{\sqrt{\omega_{cr}^4 + \omega^2 \delta^2}}; \qquad (2.17)$$

– phase shift of the static displacement, rad:

$$\varphi_s(\omega) = arctg\left(\frac{\omega\delta}{\omega_{cr}^2}\right); \qquad (2.18)$$

– amplitude-frequency response, m:

$$A_s(\omega) = \frac{\omega^2 e}{\sqrt{\left(\omega_{cr}^2 - \omega^2\right)^2 + k^2 \omega^2}}; \qquad (2.19)$$

– phase-frequency response, rad:

$$\varphi(\omega) = \begin{cases} arctg\left(\frac{k\omega}{\omega_{cr}^2 - \omega^2}\right), if\ \omega < \omega_0; \\ \frac{\pi}{2}, if\ \omega = \omega_0; \\ \pi + arctg\left(\frac{k\omega}{\omega_{cr}^2 - \omega^2}\right), if\ \omega > \omega_0; \end{cases} \qquad (2.20)$$

2) characteristics of self-oscillating precession of a floating ring considering dry friction:
– friction force, N:

$$F_f = fF_n; \qquad (2.21)$$

– partial frequency of the floating ring, rad/s:

$$\omega_r = \sqrt{\frac{c_0}{m_r}}; \qquad (2.22)$$

– precession frequency, rad/s:

$$\Omega(\omega) = 0.5\omega - \frac{F_f}{b_0 h_0 a(\omega)}, \qquad (2.23)$$

where $a(\omega)$ – the real root of the following equation:

$$\frac{3}{4}\alpha^2 \omega_0^2 a^4 - \frac{1}{4}\omega^2 a^2 + \left(\omega_r^2 + \frac{F_r}{b_0 h_0}\omega\right) a - \frac{F_r^2}{b_0^2 h_0} = 0; \quad (2.24)$$

3) the stability of a single-mass model considering the anisotropy of elastic forces:

– coefficient of the circulating force at operating speed, N/m:

$$q_0 = \frac{1}{2} b_0 \omega_0; \qquad (2.25)$$

– stiffness anisotropy, N/m:

$$\Delta c = \delta_a c_e; \qquad (2.26)$$

– maximum stiffness anisotropy, N/m:

$$\Delta c_{max} = \sqrt{c_0^2 + q_0^2}; \qquad (2.27)$$

– determination of signs of the reals parts of all the complex roots for the following characteristic equation:

$$\begin{gathered} m_r p^4 + 2b_0 m_r p^3 + (b_0^2 + 2c_0 m_r)p^2 + \\ +2b_0 c_0 p + (c_0^2 + q_0^2 - \Delta c) = 0, \end{gathered} \qquad (2.28)$$

where p – operator;

4) characteristics of self-oscillating precession of the rotor considering dry friction:

– minimum frequency, rad/s:

$$\omega_{min} = 2\frac{\Delta c}{b_0}; \qquad (2.29)$$

– auxiliary function as a semiaxes ratio for the elliptical trajectory for the phase shift of $\frac{\pi}{4}$ rad:

$$\psi(\omega) = \sqrt{\frac{\omega - \omega_{min}}{\omega + \omega_{min}}}; \qquad (2.30)$$

– precession frequency, rad/s:

$$\Omega(\omega) = 0.5\omega \frac{2\psi(\omega)}{1+\psi^2(\omega)}. \qquad (2.31)$$

Pr. tr. 7. Dynamic analysis of a discrete-mass model considering the gyroscopic moment of inertia

1. Determination of the gyroscopic moment of the disk.
2. Determination of eigenfrequencies.
3. Clarification of eigenfrequencies considering the gyroscopic moment of inertia.

The initial data is similar to Pr. 1. However, the width of the disk is additionally considered: $B = \beta L$, where β – dimensionless ratio (Table 2.3).

Table 2.3 – Additional data for Pr. tr. 7

Var.	1	2	3	4	5
β	0.005	0.010	0.015	0.020	0.025
Var.	6	7	8	9	10
β	0.030	0.035	0.040	0.005	0.010
Var.	11	12	13	14	15
β	0.015	0.020	0.025	0.030	0.035
Var.	16	17	18	19	20
β	0.040	0.005	0.010	0.015	0.020

The calculation technique is as follows:

1) determination of the gyroscopic moment of the disk:

– diameter of the disk, m:

$$D = 2\sqrt{\frac{m_0}{\pi \rho B}}; \tag{2.32}$$

– the gyroscopic moment of the disk, kg·m^2:

$$I_d = \frac{m_0 D^2}{16} - \frac{m_0 B^2}{12}; \tag{2.33}$$

2) determination of eigenfrequencies:

– calculation of the cross-sectional moment of inertia formula (1.3);

– bending stiffness of the cantilever shaft, N/m:

$$c = \frac{3EI}{L^3}; \tag{2.34}$$

– eigenfrequency of the simplified single-mass model, rad/s:

$$\omega_{cr} = \sqrt{\frac{c}{m_0}}; \tag{2.35}$$

– calculation of the shaft mass by formula (1.1);

– inertia matrix:

$$M = \begin{bmatrix} \frac{13}{35} m_s + m_0 & -\frac{11}{210} m_s L \\ -\frac{11}{210} m_s L & \frac{1}{105} m_s L^2 \end{bmatrix}; \tag{2.36}$$

– stiffness matrix:

$$C = \begin{bmatrix} 4c & -2cL \\ -2cL & \frac{4}{3}cL^2 \end{bmatrix}; \qquad (2.37)$$

– the first two eigenfrequencies ω_1, ω_2, rad/s, are determined from the following characteristic equation:

$$det(C - \omega^2 M) = 0; \qquad (2.38)$$

3) clarification of eigenfrequencies considering the gyroscopic moment of inertia:

– the compliance coefficients:

$$\delta_{11} = \frac{1}{c}; \delta_{12} = \delta_{21} = \frac{L^2}{2EI}; \delta_{22} = \frac{L}{EI}; \qquad (2.39)$$

– according to the traditional model, the first two eigenfrequencies ω_1, ω_2, rad/s, are determined from the following characteristic equation:

$$det \begin{bmatrix} m_0\delta_{11}\omega^2 - 1 & I_d\delta_{12}\omega^2 \\ I_d\delta_{21}\omega^2 & I_d\delta_{22}\omega^2 - 1 \end{bmatrix} = 0; \qquad (2.40)$$

– according to the finite element model, the first two eigenfrequencies ω_1, ω_2, rad/s, are determined from the characteristic equation (2.38), where the inertia matrix is clarified as follows:

$$M = \begin{bmatrix} \frac{13}{35}m_s + m_0 & -\frac{11}{210}m_sL \\ -\frac{11}{210}m_sL & \frac{1}{105}m_sL^2 + I_d \end{bmatrix}; \qquad (2.41)$$

Pr. tr. 8, 9. Application of the finite element method to study free and forced oscillations of rotor systems. Analysis of the finite element models of free and forced rotor oscillations using computer algebra systems

1. Determination of local matrices of inertia and stiffness.
2. Determination of global matrices of inertia and stiffness.
3. Evaluation of eigenfrequencies of free oscillations.
4. Comparison of the first eigenfrequency with the result for a single-mass model.
5. Mode shapes of free oscillations.
6. Amplitude and phase-frequency responses.
7. Shapes of forced oscillations.

The initial data is similar to Pr. tr. 1, 2, 4. The calculation technique is as follows:

1) determination of local matrices of inertia and stiffness:

– calculation of the finite element length for the system of two finite elements, m:

$$L_e = \frac{1}{2} L; \tag{2.42}$$

– calculation of shaft mass by formula (1.1);

– calculation of the finite element mass for the system of two finite elements, kg:

$$m_e = \frac{1}{2} m_s; \tag{2.43}$$

– determination of the cross-sectional moment of inertia by formula (1.3);

– local matrix of inertia for each finite element:

$$M=\begin{bmatrix} \frac{13m_e}{35} & \frac{11m_eL_e}{210} & \frac{9m_e}{70} & -\frac{13m_eL_e}{420} \\ \frac{11m_eL_e}{210} & \frac{m_eL_e^2}{105} & \frac{13m_eL_e}{420} & -\frac{m_eL_e^2}{140} \\ \frac{9m_e}{70} & \frac{13m_eL_e}{420} & \frac{13m_e}{35} & -\frac{11m_eL_e}{210} \\ -\frac{13m_eL_e}{420} & -\frac{m_eL_e^2}{140} & -\frac{11m_eL_e}{210} & \frac{m_eL_e^2}{105} \end{bmatrix}; \quad (2.44)$$

– local matrix of stiffness for each finite element:

$$C=\begin{bmatrix} \frac{12EI}{L_e^3} & \frac{6EI}{L_e^2} & -\frac{12EI}{L_e^3} & \frac{6EI}{L_e^2} \\ \frac{6EI}{L_e^2} & \frac{4EI}{L_e} & -\frac{6EI}{L_e^2} & \frac{2EI}{L_e} \\ -\frac{12EI}{L_e^3} & -\frac{6EI}{L_e^2} & \frac{12EI}{L_e^3} & -\frac{6EI}{L_e^2} \\ \frac{6EI}{L_e^2} & \frac{2EI}{L_e} & -\frac{6EI}{L_e^2} & \frac{4EI}{L_e} \end{bmatrix}; \quad (2.45)$$

2) determination of global matrices of inertia and stiffness:

$$M_g=\begin{bmatrix} M_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & M_{4,4} \end{bmatrix};\ C_g=\begin{bmatrix} C_{1,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_{4,4} \end{bmatrix}; \quad (2.46)$$

3) evaluation of eigenfrequencies of free oscillations ω_1, ω_2, and ω_3, rad/s, is realized by solving the following characteristic equation:

$$det(C_g-\omega^2 M_g)=0; \quad (2.47)$$

4) comparison of the first eigenfrequency with the result for a single-mass model:
– calculation of the equivalent mass of the rotor by formula (1.2);

– calculation of the bending stiffness of the simply-supported shaft by formula (1.4);
– determination of the equivalent stiffness of the system “rotor – bearings” by formula (1.5);
– evaluation of the eigenfrequency by formula (1.6);
5) mode shapes of free oscillations are determined using the following matrices:

$$D_1 = C_g - \omega_1^2 M_g;\ D_2 = C_g - \omega_2^2 M_g;\ D_3 = C_g - \omega_3^2 M_g; \quad (2.48)$$

6) amplitude and phase-frequency responses:
– determination of damping factor by formula (1.12);
– determination of external damping factor by formula (1.26);
– local matrix of damping:

$$C = \begin{bmatrix} \Delta b/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta b/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad (2.49)$$

– global matrix of damping:

$$B_g = \begin{bmatrix} \frac{\Delta b}{2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}; \quad (2.50)$$

– global column-vector of imbalances:

$$D_g = \begin{Bmatrix} 0 \\ 0 \\ m_0 e \\ 0 \\ 0 \\ 0 \end{Bmatrix}; \quad (2.51)$$

– global column-vector of complex displacements:

$$Y(\omega) = \left(C_g + j\omega B - \omega^2 M\right)^{-1} D_g \omega^2, \qquad (2.52)$$

where j – imaginary unit ($j^2 = -1$);
– amplitude-frequency response:

$$A(\omega) = |Y_0(\omega)_3|; \qquad (2.53)$$

– phase-frequency response:

$$A(\omega) = arg[Y(\omega)_3]\,; \qquad (2.54)$$

7) shapes of forced oscillations:

$$Y_0(\omega) = \begin{cases} |Y_0(\omega)_1|; \\ |Y_0(\omega)_3|; \\ |Y_0(\omega)_5|. \end{cases} \qquad (2.55)$$

Topic 3. Fundamentals of balancing rotors for centrifugal machines

Conditions of rotor's dynamic equilibrium. Types of unbalances. Equivalent systems of imbalances. The concept of a rigid rotor. Quality criteria in rotor balancing. Static balancing of a rotor. Dynamic balancing of a rotor. The phenomenon of unbalance for a rotor balanced in two correction planes at low frequency. The decomposition of a synchronous precession for an unbalanced rotor by mode shapes of free oscillations. Rotor balancing by mode shapes. The Den Hartog's approach in rotor balancing.

Pr. tr. 10. Static balancing of a disk

1. Calculation of trial imbalance.
2. Evaluation of unbalanced mass.
3. The quality of the balancing procedure.

The initial data is similar to Pr. tr. 1, 7. However, the following additional parameters are introduced (Table 3.1): φ_1 – disk rotation angle, °.

Table 3.1 – Additional data for Pr. tr. 10

Var.	1	2	3	4	5	6	7	8	9	10
φ_1, °	15	20	25	30	35	40	45	50	55	15
Var.	11	12	13	14	15	16	17	18	19	20
φ_1, °	20	25	30	35	40	45	50	55	15	20

The calculation technique is as follows:
1) calculation of trial imbalance:
– diameter of the disk, m:

$$D = \sqrt[3]{\frac{4m_0}{\pi\rho\beta}}; \tag{3.1}$$

– balancing speed, rad/s:

$$\omega_b = \frac{\pi n_0}{30}; \tag{3.2}$$

– balancing radius, m:

$$r_b = \frac{D}{2}; \tag{3.3}$$

– trial mass, kg:

$$m_t = 0.02\frac{m_0 g}{\omega_b^2 r_b}; \tag{3.4}$$

2) evaluation of unbalanced mass, kg:

$$m_b = \frac{m_t}{tg(\varphi_1)}; \tag{3.5}$$

3) the quality of the balancing procedure, %:

$$\delta_b = \left|1 - \frac{m_t}{m_b \, sin(\varphi_1)}\right| \cdot 100. \tag{3.6}$$

Pr. tr. 11. Dynamic balancing of the rotor in two correction planes

1. Calculation of trial imbalance.
2. Evaluation of the system of equivalent imbalances.
3. Determination of residual imbalance.
4. Calculation of the amplitude of forced oscillations.
5. Evaluation of dynamic load on bearing supports.

The initial data is similar to Pr. tr. 1, 7, 10. However, the following additional parameters are introduced (Table 3.2): A_{10}, A_{20} – amplitudes of initial displacements, m; φ_{10}, φ_{20} – phases of initial displacements, °; A_{11}, A_{21} – amplitudes of initial displacements after 1st start, m; φ_{11}, φ_{21} – phases of initial displacements after 1st start, °; A_{12}, A_{22} – amplitudes of initial displacements after 2nd start, m; φ_{12}, φ_{22} – phases of initial displacements after 2nd start, °.

Table 3.2 – Initial data for Pr. tr. 11

Var.	1–4	5–8	9–12	13–16	17–20
A_{10}, 10^{-3} m	140	145	150	155	160
φ_{10}, °	20	25	30	35	40
A_{20}, 10^{-3} m	150	155	160	165	170
φ_{20}, °	5	10	15	20	25
A_{11}, 10^{-3} m	138	143	148	153	158
φ_{11}, °	25	30	35	40	45
A_{21}, 10^{-3} m	147	152	157	162	167
φ_{21}, °	0	5	10	25	30
A_{12}, 10^{-3} m	143	148	153	158	163
φ_{12}, °	15	20	25	30	35
A_{22}, 10^{-3} m	145	150	155	160	165
φ_{22}, °	10	15	20	25	30

The calculation technique is as follows:

1) calculation of trial imbalance:

– determination of trial mass by formulas (3.1)–(3.4);

– trial imbalance, kg·m:

$$D_t = m_t r_b; \tag{3.7}$$

2) evaluation of the system of equivalent imbalances:

– initial complex amplitudes:

$$Y_{10} = A_{10} e^{\frac{\pi\varphi_{10}}{180}j};\ Y_{20} = A_{20} e^{\frac{\pi\varphi_{20}}{180}j}; \tag{3.8}$$

– complex amplitudes after 1st start:

$$Y_{11} = A_{11} e^{\frac{\pi\varphi_{11}}{180}j};\ Y_{21} = A_{21} e^{\frac{\pi\varphi_{21}}{180}j}; \tag{3.9}$$

– complex amplitudes after 2nd start:

$$Y_{12} = A_{12} e^{\frac{\pi \varphi_{12}}{180} j}; \; Y_{22} = A_{22} e^{\frac{\pi \varphi_{22}}{180} j}; \qquad (3.10)$$

– elements of the matrix of complex compliances, m/(kg·m):

$$W_{11} = \frac{Y_{11} - Y_{10}}{D_t}; \; W_{12} = \frac{Y_{12} - Y_{10}}{D_t};$$
$$W_{21} = \frac{Y_{21} - Y_{20}}{D_t}; \; W_{22} = \frac{Y_{22} - Y_{20}}{D_t}; \qquad (3.11)$$

– evaluation of complex imbalances, kg·m:

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}^{-1} \begin{Bmatrix} Y_{10} \\ Y_{20} \end{Bmatrix}; \qquad (3.12)$$

– correcting imbalances:

$$D_{b1} = -D_1; D_{b2} = -D_2; \qquad (3.13)$$

– magnitudes of the correcting imbalances, kg·m:

$$|D_{b1}| = \sqrt{Re^2(D_1) + Im^2(D_1)};$$
$$|D_{b2}| = \sqrt{Re^2(D_2) + Im^2(D_2)}; \qquad (3.14)$$

– balancing masses, kg:

$$m_{b1} = \frac{|D_{b1}|}{r_b}; \; m_{b2} = \frac{|D_{b2}|}{r_b}; \qquad (3.15)$$

– phases of the correcting masses, °:

$$\varphi_{b1} = \frac{180}{\pi} arg(D_{b1}); \; \varphi_{b2} = \frac{180}{\pi} arg(D_{b2}); \qquad (3.16)$$

3) determination of residual imbalance:

– complex residual imbalances, kg·m:

$$\Delta D_1 = D_{10} + D_{b1};\ \Delta D_2 = D_{20} + D_{b2}; \qquad (3.17)$$

– magnitudes of the residual imbalances, kg·m:

$$|\Delta D_1| = \sqrt{Re^2(\Delta D_1) + Im^2(\Delta D_1)};$$
$$|\Delta D_2| = \sqrt{Re^2(\Delta D_2) + Im^2(\Delta D_2)}; \qquad (3.18)$$

– phases of the residual imbalances, °:

$$\Delta\varphi_1 = \frac{180}{\pi} arg(\Delta D_1);\ \Delta\varphi_2 = \frac{180}{\pi} arg(\Delta D_2); \qquad (3.19)$$

4) calculation of the amplitude of forced oscillations:
– maximum amplitude before balancing, m:

$$A_{max} = \frac{max(|D_{10}|,|D_{20}|)\cdot\omega_b^2}{2c_s - m_0\omega_b^2}; \qquad (3.20)$$

– maximum amplitude after balancing, m:

$$\Delta A_{max} = \frac{max(|\Delta D_1|,|\Delta D_2|)\cdot\omega_b^2}{2c_s - m_0\omega_b^2}; \qquad (3.21)$$

5) evaluation of dynamic load on bearing supports:
– dynamic load ratio before balancing:

$$k_d = \frac{max(|D_{10}|,|D_{20}|)\cdot\omega_b^2}{m_0 g}; \qquad (3.22)$$

– dynamic load ratio after balancing:

$$\Delta k_d = \frac{max(|\Delta D_1|,|\Delta D_2|)\cdot\omega_b^2}{m_0 g}; \qquad (3.23)$$

Pr. tr. 12, 13. Virtual balancing of a rotor

1. Determination of eigenfrequencies and mode shapes of free oscillations.
2. Forced oscillations of the rotor at the operating frequency before balancing.
3. Rotor balancing by the first mode shape.
4. Balancing at the second mode shape.
5. Forced oscillations of the rotor at the operating frequency after balancing.
6. Application of the linear regression formula.

The initial data is as follows: operating speed – n_0 = 9000 rpm; density – ρ = 7850 kg/m^3; Young's modulus – E = 2.1·10^{11} N/m^2; trial imbalance – D_t = 5·10^{-4} kg·m.

The following calculation technique is realized for the simply-supported rotor by the application of the finite element analysis using the computer-algebra system:

1) determination of eigenfrequencies and mode shapes of free oscillations:

– evaluation of the first two eigenfrequencies ω_1 and ω_2, rad/s;

2) forced oscillations of the rotor at the operating frequency before balancing and choosing three correction planes:

3) rotor balancing by the first mode shape:

– evaluation of the complex amplitude on the 2nd correction plane before balancing Y_{02}, m;

– evaluation of the complex amplitude on the 2nd correction plane after balancing at the 1st mode shape Y_{12}, m;

– calculation of the phase shift for the 1st mode shape:

$$\varphi_{12} = arg(Y_{20})\,; \tag{3.24}$$

– calculation of the 1st mode shape factor:

$$\eta_{12} = \frac{Y_{02}}{Y_{12} - Y_{02}}; \quad (3.25)$$

– determination of the imbalance, kg·m:

$$D_{b12} = \eta_{12} D_t e^{\frac{\pi \varphi_{12}}{180} j}; \quad (3.26)$$

– the magnitude of the imbalance:

$$|D_{b12}| = \sqrt{Re^2(D_{b12}) + Im^2(D_{b12})}; \quad (3.27)$$

– phase of the imbalance, °:

$$\varphi_{b12} = \frac{180}{\pi} arg(D_{b12}); \quad (3.28)$$

4) balancing at the second mode shape:
– evaluation of the complex amplitude on the 1st and 3rd correction planes before balancing Y_{01}, and Y_{03}, m, respectively;
– evaluation of the complex amplitude on the 1st and 3rd correction planes after balancing at the 2nd mode shape Y_{21}, and Y_{23}, m, respectively;
– calculation of the phase shift for the 2nd mode shape:

$$\varphi_{21} = arg(Y_{01}) = arg(Y_{03}); \quad (3.29)$$

– calculation of the 2nd mode shape factors:

$$\eta_{21} = \frac{Y_{01}}{Y_{21} - Y_{01}}; \ \eta_{23} = \frac{Y_{03}}{Y_{23} - Y_{03}}; \quad (3.30)$$

– determination of the imbalances, kg·m:

$$D_{b21} = \eta_{21} D_t e^{\frac{\pi \varphi_{21}}{180} j}; \ D_{b23} = \eta_{23} D_t e^{\frac{\pi \varphi_{23}}{180} j}; \quad (3.31)$$

– magnitudes of the imbalances:

$$|D_{b21}| = \sqrt{Re^2(D_{b21}) + Im^2(D_{b21})};$$
$$|D_{b23}| = \sqrt{Re^2(D_{b23}) + Im^2(D_{b23})}; \qquad (3.32)$$

– phases of the imbalances, °:

$$\varphi_{b21} = \frac{180}{\pi} arg(D_{b21})\,; \; \varphi_{b23} = \frac{180}{\pi} arg(D_{b23}); \quad (3.33)$$

5) forced oscillations of the rotor at the operating frequency after balancing:
– evaluation of the maximum displacement after balancing, m;
6) application of the linear regression formula:
– column-vector of displacements variations, m:

$$\Delta Y = \begin{Bmatrix} Y_{i1} - Y_{01} \\ Y_{i2} - Y_{02} \\ Y_{i3} - Y_{03} \end{Bmatrix}, \qquad (3.34)$$

where Y_{01}, Y_{02}, and Y_{03} – complex displacements before balancing, m; Y_{i1}, Y_{i2}, and Y_{i3} – complex displacements at *i*-th mode shape, m;
– calculation of the *i*-th mode shape factor:

$$\eta_i = (\Delta Y^T \Delta Y)^T \Delta Y^T Y_0, \qquad (3.35)$$

where $Y_0 = \{Y_{01}, Y_{02}, Y_{03}\}^T$ – column-vector of displacements before balancing, m;
– calculation of the phase shift for the *i*-th mode shape:

$$\varphi_i = arg(Y_{01}) = arg(Y_{02}) = arg(Y_{03})\,; \qquad (3.36)$$

– determination of the imbalance, kg·m:

$$D_{bi} = \eta_i D_t e^{\frac{\pi \varphi_i}{180} j}; \qquad (3.37)$$

– the magnitude of the imbalance:

$$|D_{bi}| = \sqrt{Re^2(D_{bi}) + Im^2(D_{bi})}; \quad (3.38)$$

– phase of the imbalance, °:

$$\varphi_{bi} = \frac{180}{\pi} arg(D_{bi}). \quad (3.39)$$

Topic 4. Parameter identification of mathematical models of rotor dynamics

Simple algebraic models. A generalized algebraic model. Non-algebraic models. A single experiment. A series of experiments. An implicit model. Linear parameter identification. Linear regression formula. Balancing by the calculation model of rotor dynamics. Practical balancing of a flexible rotor on the operating frequency. Application of the linear regression formula for balancing a flexible rotor by the Den Hartog's approach. Application of the linear regression formula for balancing a flexible rotor by mode shapes.

Pr. tr. 14, 15. Practical balancing of the flexible rotor at operating frequency. Balancing of flexible rotors by Dan Hartog's approach

1. Forced oscillations of the rotor before balancing.
2. Determination of corrective imbalances.
3. Forced oscillations of the rotor after balancing.
4. Determination of amplitudes and phases of dynamic deflections after balancing.

The initial data is as follows: the 1st operating speed – n_1 = 7500 rpm; the 2nd operating speed – n_2 = 9000 rpm; density – ρ = 7850 kg/m^3; Young's modulus – $E = 2.1\cdot10^{11}$ N/m^2; trial imbalance – $D_t = 5\cdot10^{-4}$ kg·m.

The following calculation technique is realized for the simply-supported rotor by the application of the finite element analysis using the computer-algebra system:

1) forced oscillations of the rotor before balancing:

– choosing a number of correction planes ν;

– evaluation of complex displacements before balancing $Y_{a0}^{<\nu>}$, m, where a – number of measuring plane;

2) determination of corrective imbalances:

– consequent setting trial imbalances, kg·m;

– evaluation of complex displacements $Y_{a,i}^{<\nu>}$, m, where i – number of correction plane;

– evaluation of complex compliance coefficients, m/(kg·m):

$$W_{a,i}^{<\nu>} = \frac{Y_{a,i}^{<\nu>} - Y_{a,0}^{<\nu>}}{D_t}; \qquad (4.1)$$

– building matrices of complex compliance coefficients:

$$W_\nu = \left[W_{a,i}^{<\nu>}\right]; \qquad (4.2)$$

– building the total matrix of complex compliance coefficients:

$$W = [W_\nu]; \qquad (4.3)$$

– evaluation of the column-vector of imbalances, kg·m:

$$D_b = (W^T W)^{-1} W^T Y_0, \qquad (4.4)$$

where $Y_0 = \{Y_{01}, Y_{02}, Y_{03}\}^T$ – column-vector of displacements before balancing, m;

– calculation of magnitudes and phases of the imbalances by formulas (3.38) and (3.39);
– application of the corrective imbalances, kg·m;
3) forced oscillations of the rotor after balancing:
– evaluation of complex displacements after balancing $\Delta Y_a^{<\nu>}$, m;
4) determination of amplitudes and phases of dynamic deflections after balancing:
– amplitudes of the complex displacements after balancing:

$$|\Delta Y_a^{<\nu>}| = \sqrt{Re^2(\Delta Y_a^{<\nu>}) + Im^2(\Delta Y_a^{<\nu>})}; \qquad (4.5)$$

– phases of the complex displacements after balancing, °:

$$\Delta\varphi_a = \frac{180}{\pi} arg(\Delta Y_a^{<\nu>}). \qquad (4.6)$$

Pr. tr. 16. Nonlinear oscillations of a rotor

1. Estimation of discrete masses.
2. Harmonic analysis of a nonlinear discrete model.
3. Dynamic stability of the rotor.

The initial data is density ρ = 7850 kg/m^3, and Young's modulus E = 2.1·10^{11} N/m^2.

The following calculation technique is realized for the simply-supported rotor by the application of the finite element analysis using the computer-algebra system:
1) estimation of discrete masses:
– choosing a number of single masses N;
– evaluation of the complex compliance coefficients $\delta_{i,l}$ (i, l = 1, 2, …, N), m/N;
– evaluation of the first N eigenfrequencies ω_ν (ν = 1, 2, …, N);

– evaluation of the first *N* mode shapes $U_i^{<\nu>}$;
– evaluation of the auxiliary parameters, s²:

$$z_\nu = \frac{1}{\omega_\nu^2}; \tag{4.7}$$

– evaluation of the following components:

$$K_{i,j}^{<\nu>} = \delta_{i,l} U_l^{<\nu>}; C_i^{<\nu>} = z_\nu U_i^{<\nu>}; \tag{4.8}$$

– forming of the following matrix and column-vector:

$$K = [K^{<\nu>}]; C = [C^{<\nu>}]; \tag{4.9}$$

– evaluation of the column-vector of discrete masses:

$$m = (K^T K)^{-1} K^T C; \tag{4.10}$$

2) harmonic analysis of a nonlinear discrete model:
– formation of the system of nonlinear equations for rotor's forced oscillations:

$$\begin{cases} x_i = \sum_{l=1}^{N} \delta_{i,l} F_{lx}; \\ y_i = \sum_{l=1}^{N} \delta_{i,l} F_{ly}, \end{cases} \tag{4.11}$$

where F_{lx}, F_{ly} – components of linear and nonlinear forces, N;
3) dynamic stability of the rotor is checked by the Routh–Hurwitz criteria.

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